IEOR 240 LP Formulation Practice

This example is adapted from *Introduction to Linear Optimization* by Bertsimas and Tsitsiklis

Power Generation

The state of California would like to plan its power generation capacity for the next 50 years. We know the following:

1. There will be a power demand of d_t MWs in year t
2. Current power plants will provide e MWs of power in each year
3. The state has the option to build two types of power plants:
   a) Solar arrays, which cost s_t per MW to build in year t and last for 10 years
   b) Nuclear plants, which cost n_t per MW to build in year t and last for 20 years

How can California formulate a linear program to find the cheapest way to meet the demand forecast? You may assume once the plant is built there are no ongoing upkeep or variable costs.

Decision Variables

What decision variables do we need to define?

Decision Variables

What decision variables do we need to define?

x_t = the number of MW of solar arrays to build in year t

y_t = the number of MW of nuclear plants to build in year t

Objective Function

We need to minimize the cost of meeting the power demand forecast

Objective Function

We need to minimize the cost of meeting the power demand forecast

$$\sum_{t=1}^{50} s_t x_t + n_t y_t$$

Constraint: Meet the demand forecast

Does this work?

$$x_t + y_t + e \geq d_t$$

Constraint: Meet the demand forecast

Does this work?

$$x_t + y_t + e \geq d_t$$

NO! Why not?

Constraint: Meet the demand forecast

Does this work?

$$x_t + y_t + e \geq d_t$$

NO! Why not?

This is only counting the power **built** in year t and is not giving any credit for the many years the plant will continue functioning

New Auxiliary Decision Variables

We need to create new decision variables to account for the total power **available** in year t

w_t = the number of MWs from solar arrays available in year t

z_t = the number of MWs from nuclear plants available in year t

Constraint: Define w_t and z_t

We can define w_t and z_t based on the preceeding values of x_t and y_t

Constraint: Define w_t and z_t

We can define w_t and z_t based on the preceeding values of x_t and y_t

$$w_t = \sum_{k=?}^{?} x_k$$

Constraint: Define w_t and z_t

We can define w_t and z_t based on the preceeding values of x_t and y_t

$$w_t = \sum_{k=(t-9)}^{t} x_k$$

Constraint: Define w_t and z_t

We can define w_t and z_t based on the preceeding values of x_t and y_t

$$w_t = \sum_{k=(t-9)}^{t} x_k$$

$$z_t = \sum_{k=(t-19)}^{t} y_k$$

But there is still one small problem...

Constraint: Define w_t and z_t

We can define w_t and z_t based on the preceeding values of x_t and y_t

$$w_t = \sum_{k=(t-9)}^{t} x_k$$

$$z_t = \sum_{k=(t-19)}^{t} y_k$$

But there is still one small problem...

What if $t - 19 < 1$?

Constraint: Define w_t and z_t

We can define w_t and z_t based on the preceeding values of x_t and y_t

$$w_t = \sum_{k=\max(1,(t-9))}^{t} x_k$$

$$z_t = \sum_{k=\max(1,(t-19))}^{t} y_k$$

Constraint: Meet the demand forecast

Now we can write the demand constraint in terms of these new variables:

$$w_t + z_t + e \geq d_t$$

Full formulation

x_t = the number of MW of solar arrays to build in year t

y_t = the number of MW of nuclear plants to build in year t

w_t = the number of MWs from solar arrays available in year t

z_t = the number of MWs from nuclear plants available in year t

$$
\begin{array}{lll}
\min & \sum_{t=1}^{50} s_t x_t + n_t y_t & \\
\text{s.t.} & w_t + z_t + e \geq d_t & \text{for } t = 1...50 \\
 & w_t = \sum_{k=\max(1,(t-9))}^{t} x_k & \text{for } t = 1...50 \\
 & z_t = \sum_{k=\max(1,(t-19))}^{t} y_k & \text{for } t = 1...50 \\
 & x_t, y_t, w_t, z_t \geq 0 & \text{for } t = 1...50
\end{array}
$$

New Constraint: Limiting nuclear power

Suppose people in California are afraid of too many nuclear power plants, so the state decides to limit the total amount of MW that can be made from nuclear at any one time to never exceed 20%. How can you add this new constraint to the model?

New Constraint: Limiting nuclear power

Suppose people in California are afraid of too many nuclear power plants, so the state decides to limit the total amount of MW that can be made from nuclear at any one time to never exceed 20%. How can you add this new constraint to the model?

Here is one idea:

$$\frac{z_t}{w_t + z_t + e} \leq 0.2$$

New Constraint: Limiting nuclear power

Suppose people in California are afraid of too many nuclear power plants, so the state decides to limit the total amount of MW that can be made from nuclear at any one time to never exceed 20%. How can you add this new constraint to the model?

Here is one idea:

$$\frac{z_t}{w_t + z_t + e} \leq 0.2$$

What is wrong with this constraint?

New Constraint: Limiting nuclear power

Suppose people in California are afraid of too many nuclear power plants, so the state decides to limit the total amount of MW that can be made from nuclear at any one time to never exceed 20%. How can you add this new constraint to the model?

Here is one idea:

$$\frac{z_t}{w_t + z_t + e} \leq 0.2$$

What is wrong with this constraint?

It is not linear

New Constraint: Limiting nuclear power

Luckily, we can easily make it linear:

$$z_t \leq 0.2(w_t + z_t + e)$$

or

$$0.8 z_t \leq 0.2(w_t + e)$$

Full formulation: Add Limiting nuclear power

x_t = the number of MW of solar arrays to build in year t

y_t = the number of MW of nuclear plants to build in year t

w_t = the number of MWs from solar arrays available in year t

z_t = the number of MWs from nuclear plants available in year t

$$
\begin{array}{lll}
\min & \sum_{t=1}^{50} s_t x_t + n_t y_t & \\
\text{s.t.} & w_t + z_t + e \geq d_t & \text{for } t = 1...50 \\
& w_t = \sum_{k=\max(1,(t-9))}^{t} x_k & \text{for } t = 1...50 \\
& z_t = \sum_{k=\max(1,(t-19))}^{t} y_k & \text{for } t = 1...50 \\
& x_t, y_t, w_t, z_t \geq 0 & \text{for } t = 1...50 \\
& 0.8 z_t \leq 0.2(w_t + e) & \text{for } t = 1...50
\end{array}
$$

New Constraint: Shutting down old power plants

Remember that we said there is e power being generated already? Lets assume that this is all being produced by coal plants that are bad for the enviornment. Now say that California would get a dollars from the federal governement if it shut down all of the coal plants being used, and that it would get the equivalent percentage if only some of the coal plants were shut down. For instance, if they shut down 10% of the coal plants they get 10% of a dollars.

How would you add this to the model? Assume that once the coal plant is shut down it cannot be used again.

Decsion Varaiable

v_t = the **percent** of MWs of coal plants shut down **by** year t

Decsion Varaiable

v_t = the **percent** of MWs of coal plants shut down **by** year t

Now we can rewrite the constraint as

$$w_t + z_t + (1 - v_t)e \geq d_t$$

$$0 \leq v_t \leq 1$$

Is this linear?

Decsion Varaiable

v_t = the **percent** of MWs of coal plants shut down **by** year t

Now we can rewrite the constraint as

$$w_t + z_t + (1 - v_t)e \geq d_t$$

$$0 \leq v_t \leq 1$$

Is this linear?

Yes, because e is a parameter not a variable

One more contraint

How do we prevent the model from re-opening closed coal plants?

For instance, it could set $v_1 = 0.9$ and $v_2 = 1$ which should not be allowed.

One more contraint

How do we prevent the model from re-opening closed coal plants?
For instance, it could set $v_1 = 0.1$ and $v_2 = 0$ which should not be allowed.

$$v_t \leq v_{t+1}$$

This will ensure a shut down plant stays shut down

New objective function

How do we add the benefit from shutting down these plants to the model?

Remember, California only gets the money **once**.

New objective function

How do we add the benefit from shutting down these plants to the model?

Remember, California only gets the money **once**.

$$-av_{50} + \sum_{t=1}^{50} s_t x_t + n_t y_t$$

Why is it negative?

New objective function

How do we add the benefit from shutting down these plants to the model?

Remember, California only gets the money **once**.

$$-av_{50} + \sum_{t=1}^{50} s_t x_t + n_t y_t$$

Why is it negative?
Because we are minimizing costs, and this is money that the state is getting, not spending.

Full formulation: Add Shutting Down Old Plants

x_t = the number of MW of solar arrays to build in year t

y_t = the number of MW of nuclear plants to build in year t

w_t = the number of MWs from solar arrays available in year t

z_t = the number of MWs from nuclear plants available in year t

v_t = the **percent** of MWs of coal plants shut down **by** year t

$$
\begin{array}{lll}
\min & -av_{50} + \sum_{t=1}^{50} s_t x_t + n_t y_t & \\
\text{s.t.} & w_t + z_t + e \geq d_t & \text{for } t = 1...50 \\
& w_t = \sum_{k=\max(1,(t-9))}^{t} x_k & \text{for } t = 1...50 \\
& z_t = \sum_{k=\max(1,(t-19))}^{t} y_k & \text{for } t = 1...50 \\
& x_t, y_t, w_t, z_t \geq 0 & \text{for } t = 1...50 \\
& 0.8 z_t \leq 0.2(w_t + e) & \text{for } t = 1...50 \\
& w_t + z_t + (1 - v_t)e \geq d_t & \text{for } t = 1...50 \\
& 0 \leq v_t \leq 1, \quad v_t \leq v_{t+1} & \text{for } t = 1...50
\end{array}
$$