Simplex method

Fall 2021

Overview

Simplex-method

Reminder: linearization of MinMax

Sensitivity analysis

Farkas Lemma: Geometrical picture

Idea: certificate of optimality, expanded

(Primal) feasibility

$$A\mathbf{x} \geq \mathbf{b},$$
$$\mathbf{x} \geq 0.$$

Boundedness (aka Dual feasibility)

$$\mathbf{y}^T A \leq \mathbf{c},$$
$$\mathbf{y} \geq 0.$$

Tightness:

$$\mathbf{c}^\top \mathbf{x} = \mathbf{y}^\top \mathbf{b}$$

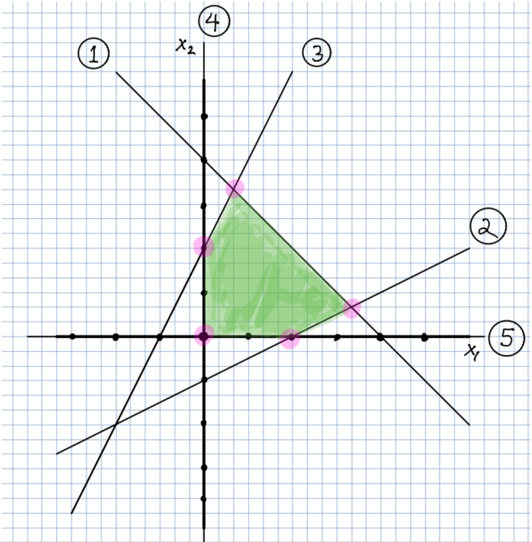
Tightness can be replaced with the **Complementary Slackness Condition**

$$[\mathbf{c}^\top - \mathbf{y}^\top A]_i x_i = 0 \text{ for all } i \in \{1 \dots n\} \text{ (a)}$$

$$[Ax - \mathbf{b}]_j y_j = 0 \text{ for all } j \in \{1 \dots m\} \text{ (b)}$$

Fact: The optimum can be achieved only in an extreme feasible point. (This is a general fact for any concave objective)
Idea behind Simplex Method: Walk on (x, y) of the form (feasible extreme point, dual point) satisfying complementary slackness, looking for y that satisfies dual feasibility

Example



$$
\begin{array}{lllcl}
\min & -x_1 & -2x_2 & & \\
\text{s.t.} & -x_1 & -x_2 & \geq & -4 \\
 & -x_1 & +2x_2 & \geq & -2 \\
 & 2x_1 & -x_2 & \geq & -2 \\
 & x_1 & & \geq & 0 \\
 & & x_2 & \geq & 0
\end{array}
$$

Finding a basic solution

Example:

$$A = \begin{pmatrix} 3 & -2 & 1 & 4 \\ 0 & 1 & 3 & -6 \\ -1 & 2 & 2 & 0 \\ 3 & -1 & -1 & 2 \\ 6 & 0 & 3 & -2 \end{pmatrix}, \quad b = \begin{pmatrix} 5 \\ 8 \\ 3 \\ 1 \\ 15 \end{pmatrix}, \quad c = \begin{pmatrix} 5 \\ 0 \\ -2 \\ 1 \end{pmatrix}$$

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}, \quad y = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{pmatrix}$$

$\min c^T x$	$\max b^T y$
$Ax \geq b$	s.t $A^T y \leq c$
s.t $x \geq 0$	$y \geq 0$

Finding a basic solution

$$I = \{1, 3\}, \quad J = \{2, 3\}, \quad \hat{I} = \{2, 4, 5\}, \quad \hat{J} = \{1, 4\}$$

$$A_{IJ} = \begin{pmatrix} -2 & 1 \\ 2 & 2 \end{pmatrix} \qquad \bar{x}_J = \begin{pmatrix} \bar{x}_2 \\ \bar{x}_3 \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ 2 & 2 \end{pmatrix}^{-1} \begin{pmatrix} 5 \\ 3 \end{pmatrix}$$

$$\bar{x}_{\hat{J}} = \begin{pmatrix} \bar{x}_1 \\ \bar{x}_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\bar{y}_I = \begin{Bmatrix} \bar{y}_1 \\ \bar{y}_3 \end{Bmatrix} = \begin{pmatrix} -2 & 2 \\ 1 & -2 \end{pmatrix}^{-1} \begin{pmatrix} 5 \\ -2 \end{pmatrix}, \qquad \bar{y}_{\hat{I}} = \begin{pmatrix} \bar{y}_2 \\ \bar{y}_4 \\ \bar{y}_5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\bar{s}_I = \begin{pmatrix} \bar{s}_1 \\ \bar{s}_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \qquad \bar{s}_{\hat{I}} = \begin{pmatrix} 1 & 3 \\ -1 & -1 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} \bar{x}_2 \\ \bar{x}_3 \end{pmatrix} - \begin{pmatrix} 8 \\ 1 \\ 15 \end{pmatrix}$$

$$\bar{r}_J = \begin{pmatrix} \bar{r}_2 \\ \bar{r}_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \qquad \bar{r}_{\hat{J}} = \begin{pmatrix} \bar{r}_1 \\ \bar{r}_4 \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 & 3 \\ 2 & 2 \end{pmatrix}^{-1} \begin{pmatrix} \bar{y}_2 \\ y_3 \end{pmatrix}$$

Finding a basic solution

Can use Gauss-Jordan method to calculate the inverse

$$A_{IJ}^{-1} = \begin{pmatrix} -2 & 1 \\ 2 & 2 \end{pmatrix}^{-1} = \frac{1}{6}\begin{pmatrix} -2 & 1 \\ 2 & 2 \end{pmatrix}$$

		5
		3
-2	1	-7
2	2	16

$$\bar{x}_J = \begin{pmatrix} \bar{x}_2 \\ \bar{x}_3 \end{pmatrix} = A_{IJ}^{-1}\underbrace{\begin{pmatrix} b_1 \\ b_3 \end{pmatrix}}_{= b_I} = \frac{1}{6}\begin{pmatrix} -2 & 1 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} 5 \\ 3 \end{pmatrix} = \frac{1}{6}\begin{pmatrix} -7 \\ 16 \end{pmatrix}$$

Another way: $\begin{cases} -2x_2 + x_3 = 5 \\ 2x_2 + 2x_3 = 3 \end{cases} \Rightarrow 3x_3 = 8 \Rightarrow -6x_2 + 8 = 15$

$$6x_3 = 16 \qquad -6x_2 = 7$$

$$\Downarrow$$

$$\begin{pmatrix} \bar{x}_2 \\ \bar{x}_3 \end{pmatrix} = \frac{1}{6}\begin{pmatrix} -7 \\ 16 \end{pmatrix}$$

c.

Finding a basic solution

$$\bar{y}_I = \begin{pmatrix}\bar{y}_1\\ \bar{y}_3\end{pmatrix} = \left(A_{IJ}^{-1}\right)^T \underbrace{\begin{pmatrix}c_2\\ c_3\end{pmatrix}}_{=c_J} = \frac{1}{6}\begin{pmatrix}-2 & 2\\ 1 & 2\end{pmatrix}\begin{pmatrix}0\\ -2\end{pmatrix} = \frac{1}{6}\begin{pmatrix}-4\\ -4\end{pmatrix} = -\frac{2}{3}\begin{pmatrix}1\\ 1\end{pmatrix}$$

$$\bar{s}_{\hat{I}} = \begin{pmatrix}\bar{s}_2\\ \bar{s}_4\\ \bar{s}_5\end{pmatrix} = A_{\hat{I}J}\bar{x}_J - b_{\hat{I}} = \frac{1}{6}\begin{pmatrix}1 & 3\\ -1 & -1\\ 0 & 3\end{pmatrix}\begin{pmatrix}-7\\ 16\end{pmatrix} - \begin{pmatrix}8\\ 1\\ 15\end{pmatrix} = \frac{1}{6}\begin{pmatrix}41\\ -9\\ 48\end{pmatrix} - \frac{1}{6}\begin{pmatrix}48\\ 6\\ 90\end{pmatrix} = -\frac{1}{6}\begin{pmatrix}7\\ 15\\ 42\end{pmatrix}$$

$$\bar{r}_{\hat{J}} = \begin{pmatrix}\bar{r}_1\\ \bar{r}_4\end{pmatrix} = c_{\hat{J}} - A_{I\hat{J}}^T\bar{y}_I = \begin{pmatrix}5\\ 1\end{pmatrix} - \left(-\frac{2}{3}\right)\begin{pmatrix}3 & -1\\ 4 & 0\end{pmatrix}\begin{pmatrix}1\\ 1\end{pmatrix} = \frac{1}{3}\begin{pmatrix}15\\ 3\end{pmatrix} + \frac{1}{3}\begin{pmatrix}4\\ 8\end{pmatrix} = \frac{1}{3}\begin{pmatrix}19\\ 11\end{pmatrix}$$

General linearization of MaxMin (or MinMax)

Similarly, for $g(y, \mathbf{x})$ increasing with y,

$$\min_{\mathbf{x} \in \mathcal{X}} \quad g(\max\{f_1(\mathbf{x}), \ldots, f_m(\mathbf{x})\}, \mathbf{x})$$

is equivalent (in some sense) to

$$\begin{aligned} \min_{\mathbf{x} \in \mathcal{X}} \quad & g(z, \mathbf{x}) \\ \text{s.t.} \quad & z \geq f_i(\mathbf{x}) \qquad \forall i \in \{1, \ldots, m\} \end{aligned}$$

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Note also that

$$\max f(x) = -\min(-f(x))$$

Example

Suppose we have the problem:

$$\min_{x_1,x_2} |x_1 + 5x_2|$$
$$\text{s.t. } x_1 - 3x_2 \geq 2$$
$$x_1 \geq 0$$

How can we convert it into a Linear Program?
Remember $|x| = \max\{x, -x\}$.

$$\min_{x_1,x_2} \quad \max\{x_1 + 5x_2, -x_1 - 5x_2\}$$
$$\text{s.t. } x_1 - 3x_2 \geq 2$$
$$x_1 \geq 0$$

Form 1

Create a new variable z and make:

$$\begin{aligned} \min_{x_1, x_2, z} \ & z \\ \text{s.t.} \ z &\geq x_1 + 5x_2 \\ z &\geq -(x_1 + 5x_2) \\ x_1 - 3x_2 &\geq 2 \\ x_1 &\geq 0 \end{aligned}$$

Terminology

- **Shadow price** y_i = Dual variable
  Change of the objective function from one unit **increase** in its right-hand side b_i
- **Reduced cost** r_j = Dual slack = $(\mathbf{c}^\top - \mathbf{y}^\top A)_j$
  Amount by which the cost coefficient of non-basic variable c_j must be **lowered** for that variable to become basic
- **Allowable increase/decrease**
  - Optimal solution x^* and objective $\sum_{j=1}^{n} c_j x_j^*$ may change
  - Whether a decision variable is basic or non-basic stays unchanged
  - Whether a constraint is binding or non-binding stays unchanged

Terminology

For a problem in symmetrical form, let $(\bar{\mathbf{x}}, \bar{\mathbf{y}})$ be primal-dual feasible point satisfying complementary slackness. Let $\bar{\mathbf{s}}$ be corresponding primal slack and $\bar{\mathbf{r}}$ be corresponding dual slack.

- Decision variable $\bar{x}_j$ is **basic** if $\bar{x}_j \neq 0$ ($\bar{r}_j = 0$ due to CS)
- Decision variable $\bar{x}_j$ is **non-basic** if $\bar{x}_j = 0$ ($\bar{r}_j < 0$ in general)
- Constraint $\sum_{j=1}^{n} a_{ij} x_j \geq b_i$ is **binding** if $\sum_{j=1}^{n} a_{ij} \bar{x}_j = b_i$ ($\bar{s}_i = 0$)
- Constraint $\sum_{j=1}^{n} a_{ij} x_j \geq b_i$ is **not binding** if $\sum_{j=1}^{n} a_{ij} \bar{x}_j > b_i$ ($\bar{y}_i = 0$ due to CS)

Example: continuous knapsack

```
ampl: include cont_knapsack.run;
CPLEX 12.6.1.0: sensitivity
CPLEX 12.6.1.0: optimal solution; objective 85
1 dual simplex iterations (1 in phase I)

suffix up OUT;
suffix down OUT;
suffix current OUT;
:    x         x.rc       x.current    x.down     x.up       :=
1   2.5    -3.55271e-15      24           20     1e+20
2   5      -1.77636e-15       5       -1e+20        12
3   0      -0.4               2       -1e+20         2.4
4   0     -15                 3       -1e+20        18
;

: _conname   _con  _con.slack _con.current _con.down     _con.up      :=
1   volume    0       12.5          60         47.5      1e+20
2   weight    1.2      0           100         50          183.333
3   water    -7        0             5          0            6.92308
;
```

AMPL notation

- `x` – primal variable;
- `x.rc` – reduced cost or dual slack;
- `x.current` – objective coefficients ($\mathbf{c}_i$);
- `_conname` – shadow price or dual variable;
- `_con.slack` – primal slack;
- `_con.current` – right hand side ($\mathbf{b}_j$);
- `...down` and `...up` are the minimal and the maximal value of the corresponding parameter $\mathbf{c}_i$ or $\mathbf{b}_j$ such that the problem stays within the allowable increase/decrease range

How to derive sensitivity analysis: Key Idea

In order for a change to be withing the allowable range, both of these must be true at the solution point:

- Whether a decision variable is basic or non-basic stays unchanged.
- Whether a constraint is binding or non-binding stays unchanged.

Types of analysis

- Case 1: Change b_i
  - Case 1a: Change b_i of non-binding constraint
  - Case 1b: Change b_i of binding constraint
  - Case 1c: Find g if Case 1b.
- Case 2: Change c_j
  - Case 2a: Change c_j of non-basic variable
  - Case 2b: Change c_j of basic variable
- Case 3: Change a_{ij}
  - Case 3a: Change a_{ij} of non-basic variable
  - *Case 3b: Change a_{ij} of basic variable*
- Case 4: Add a new constraint
- Case 5: Add a new decision variable

Case 2a: Change c_j of non-basic variable

Change c_j of non-basic variable

- Reduced cost $r_j \neq 0$ [1]

$$r_j = c_j - \sum_{i=1}^{m} a_{ij} y_i$$

- Consider c_3 which has reduced cost $r_3 = -0.4$
  - Allowable increase: $-r_j = 0.4$
  - Allowable decrease: $+\infty$
- Consider changing c_3 from 2 to 2.1
  - New optimal solution: Unchanged
  - New optimal objective value: Unchanged
- It's possible to change several c_j for non-basics variables at the same time!

[1] $r_j = 0$ for non-basic variable means multiple optimal solutions

Case 2b: Change c_j of basic variable

Change c_j of basic variable

$$
\begin{array}{rrrrrcl}
\max & 24x_1 & +(5+\delta)x_2 & +2x_3 & +3x_4 & & \\
s.t. & 3x_1 & +8x_2 & +14x_3 & +6x_4 & \leq & 60 \\
 & 20x_1 & +10x_2 & +2x_3 & +15x_4 & \leq & 100 \\
 & & x_2 & & & \geq & 5 \\
 & x_1 & & & & \geq & 0 \\
 & & x_2 & & & \geq & 0 \\
 & & & x_3 & & \geq & 0 \\
 & & & & x_4 & \geq & 0
\end{array}
$$

Case 2b: Change c_j of basic variable

Consider the Dual problem:

$$
\begin{array}{lccccccr}
\min & 60y_1 & +100y_2 & +5y_3 & & & \\
s.t. & 3y_1 & +20y_2 & & \geq & 24 & (1) \\
 & 8y_1 & +10y_2 & +y_3 & \geq & 5+\delta & (2) \\
 & 14y_1 & +2y_2 & & \geq & 2 & (3) \\
 & 6y_1 & +15y_2 & & \geq & 3 & (4) \\
 & y_1 & & & \geq & 0 & (5) \\
 & & y_2 & & \geq & 0 & (6) \\
 & & & y_3 & \leq & 0 & (7)
\end{array}
$$

Optimal solution $y_1 = 0, y_2 = 1.2, y_3 = -7$

Case 2b: Change c_j of basic variable

$$
\begin{array}{llllll}
\min & +100y_2 & +5y_3 & & & \\
s.t. & +20y_2 & & = & 24 & (1) \\
 & +10y_2 & +y_3 & = & 5+\delta & (2) \\
 & +2y_2 & & \geq & 2 & (3) \\
 & +15y_2 & & \geq & 3 & (4) \\
 & y_2 & & \geq & 0 & (6) \\
 & & y_3 & \leq & 0 & (7)
\end{array}
$$

Case 2b: Change c_j of basic variable

From (1) we get $y_2 = \frac{24}{20}$ (satisfies (3), (4), (6)), substitute in (2)

$$y_3 = -7 + \delta.$$

From (7)

$$\delta \leq 7$$

Case 2b: Change c_j of basic variable

Change c_j of basic variable

- Reduced cost $r_j = 0$
- Consider c_2
  - Allowable increase: 7
  - Allowable decrease: $+\infty$
- Consider changing c_2 from $5 \to 10$
  - New optimal solution: Unchanged
  - New optimal objective value:

$$\sum_{j=1}^{n} c_j^{new} x_j^* = \sum_{j=1}^{n} c_j x_j^* + \delta x_2^* = 110$$

Case 4: Add a new constraint

Add a new constraint

- If current solution satisfies the new constraint
  - New optimal solution: Unchanged
  - New optimal objective value: Unchanged
- If current solution does not satisfy the new constraint
  - Dual simplex method (but don't worry about this for now)

Note: the problem might become infeasible

Note

Sensitivity analysis lets you simultaniously think about a continious set of instances of LP for which δ is within the range. The other instances still have to be considered individually.

Farkas Lemma formulation (for Standard form)

Standard form LP:

$$\begin{aligned} \min \quad & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} \quad & A\mathbf{x} = \mathbf{b}, \\ & \mathbf{x} \geq 0. \end{aligned}$$

Feasibility certificate:

$$\begin{aligned} A\mathbf{x} &= \mathbf{b}, \\ \mathbf{x} &\geq 0. \end{aligned}$$

Infeasibility certificate:

$$\begin{aligned} \mathbf{y}^T \mathbf{b} &> 0 \\ \mathbf{y}^T A &\leq 0. \end{aligned}$$

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Farkas Lemma: Exactly one out of two exists: x or y
(Equivalent to Theorem 2 of alternatives from LPMATH)

Reminder from Linear Algebra

Consider $\mathbf{a}_1, \ldots, \mathbf{a}_n \in \mathbb{R}^m$, $\mathbf{y} \in \mathbb{R}^m$ and $\mathbf{x} \in \mathbb{R}^n$. The matrix $[\mathbf{a}_1 \ldots \mathbf{a}_n] = A \in \mathbb{R}^{m \times n}$

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- Columns of the matrix in multiplication on the left:

$$\mathbf{y}^\top A = \mathbf{y}^\top [\mathbf{a}_1 \dots \mathbf{a}_n] = [\mathbf{y}^\top \mathbf{a}_1 \ \dots \ \mathbf{y}^\top \mathbf{a}_n]$$

Reminder from Linear Algebra

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- Columns of the matrix in multiplication on the left:

$$\mathbf{y}^\top A = \mathbf{y}^\top [\mathbf{a}_1 \ldots \mathbf{a}_n] = [\mathbf{y}^\top \mathbf{a}_1 \;\; \ldots \;\; \mathbf{y}^\top \mathbf{a}_n]$$

- Columns of the matrix in multiplication on the right:

$$A\mathbf{x} = [\mathbf{a}_1 \ldots \mathbf{a}_n] \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = x_1 \mathbf{a}_1 + \ldots + x_n \mathbf{a}_n = \sum_{j=1}^{n} x_j \mathbf{a}_j$$

Conic combination of vectors

For $\mathbf{a}_1, \ldots, \mathbf{a}_m \in \mathbb{R}^n$, a *linear combination* is a vector $\mathbf{v} \in \mathbb{R}^n$ that can be represented as

$$\mathbf{v} = \sum_{i=1}^{m} w_i \mathbf{a}_i \text{ or } v = \mathbf{v}^\top A$$

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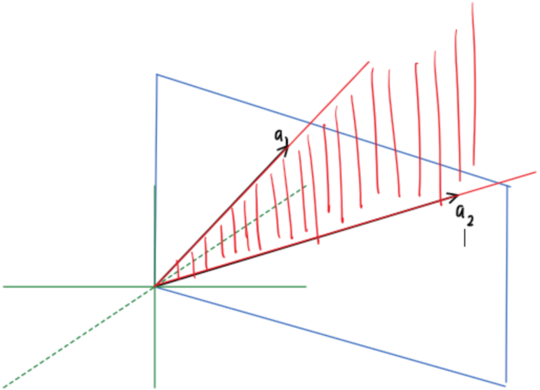
for some $w_1, \ldots, w_m \geq 0$

Conic combination of vectors

As the set of all linear combinations produces a linear span, the set of all conic combinations produces a **cone**.



Geometric interpretation

Standard form LP:

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Feasibility certificate:

$$A\mathbf{x} = \mathbf{b},$$
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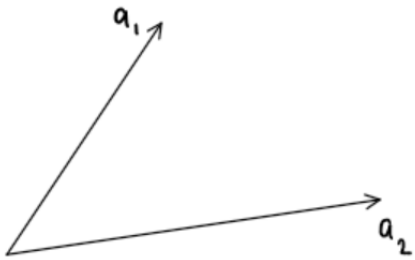
Infeasibility certificate:

$$\mathbf{y}^T\mathbf{b} > 0$$
$$\mathbf{y}^T A \leq 0.$$



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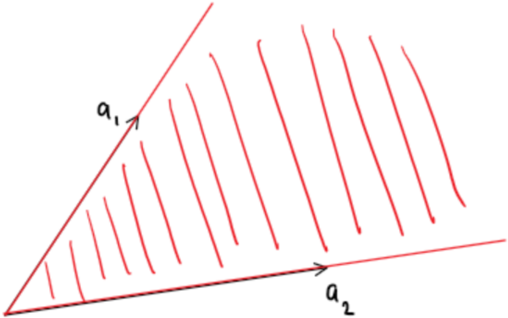

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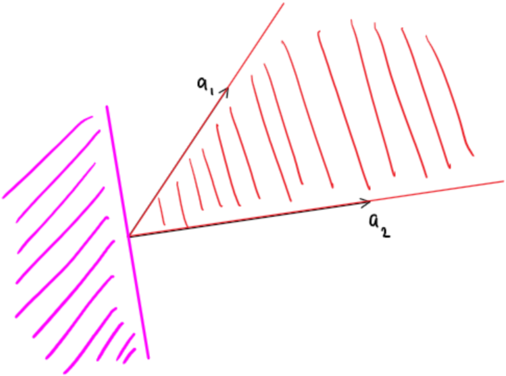

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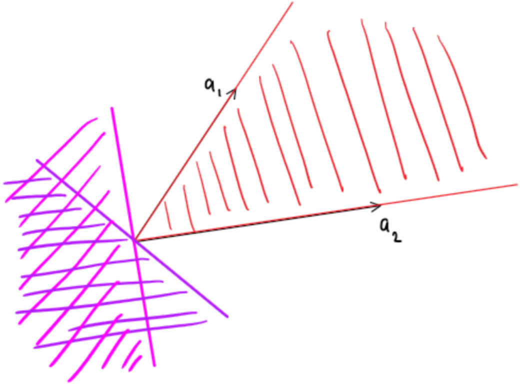

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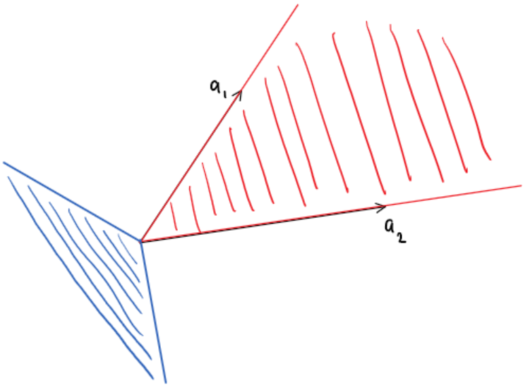

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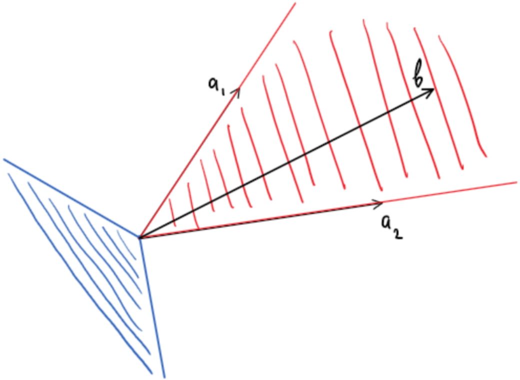


Geometric interpretation

Standard form LP:

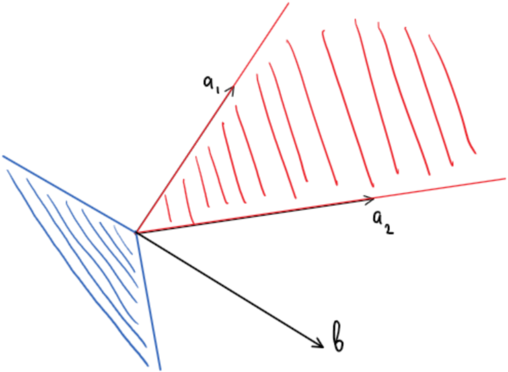
$$\begin{aligned} \min \quad & \mathbf{c}^T\mathbf{x} \\ \text{s.t.} \quad & A\mathbf{x} = \mathbf{b}, \\ & \mathbf{x} \geq 0. \end{aligned}$$

Feasibility certificate:

$$\begin{aligned} A\mathbf{x} &= \mathbf{b}, \\ \mathbf{x} &\geq 0. \end{aligned}$$

Infeasibility certificate:

$$\begin{aligned} \mathbf{y}^T\mathbf{b} &> 0 \\ \mathbf{y}^T A &\leq 0. \end{aligned}$$



Thank you for your attention !