EE650 Linear System Theory
Problem Set 3

Issued 15 Oct 2023; Due 17 Nov 2023 14:00 HST

Problem 1

Consider the matrices:

$$A_1 = \begin{bmatrix} 6 & 5 & -1 \\ -1 & 0 & 1 \\ 3 & 3 & 2 \end{bmatrix} \quad A_2 = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 5 & -2 \\ 3 & 4 & 0 \end{bmatrix}$$

By defining an appropriate similarity transform in each case, put each matrix into either diagonal or Jordan form.

Problem 2

Suppose $A \in \mathbb{C}^{n\times n}$ is such that $\det(A) = 0$. Is $\det(e^A) = 0$? Explain why or why not.

Problem 3

A matrix $A \in \mathbb{R}^{6\times 6}$ has minimal polynomial s^3. Give bounds on the rank of A.

Problem 4

A matrix A has a minimal polynomial $(s-\lambda_1)^2(s-\lambda_2)^3$. Find $\cos(e^A)$ as a polynomial in A.

Problem 5

A matrix A has a minimal polynomial $(s-\lambda_1)^2(s-\lambda_2)^3$, a characteristic polynomial $(s-\lambda_1)^5(s-\lambda_2)^3$ and has four linearly independent eigenvectors. Write down the Jordan form J of this matrix and write down $\cos(e^A)$ explicitly.

Problem 6

Let $A \in \mathbb{R}^{n\times n}$ be non-singular. True or false: the nullspace of $\cos(\log(A))$ is an $A-$invariant subspace.

Problem 7

Consider $A \in \mathbb{R}^{n\times n}, b \in \mathbb{R}^n$. Show that $\text{span}\{b, Ab, \ldots, A^{n-1}b\}$ is an $A-$invariant subspace.

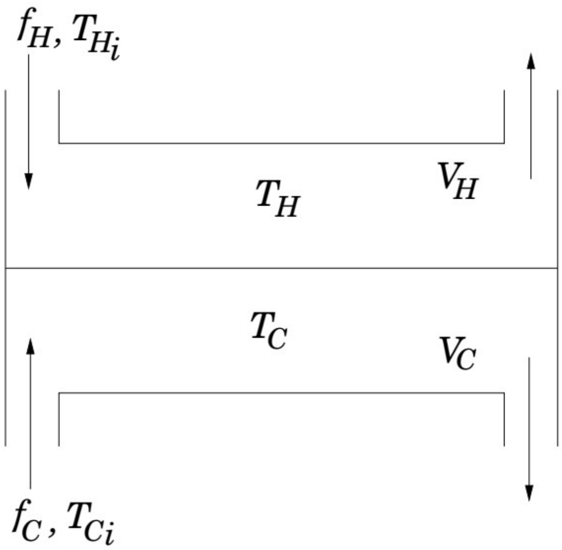

Figure 1: A simple heat exchanger

Problem 8

BIBO Stability. Consider the simple heat exchanger shown in Figure , in which f_C and f_H are the flows (assumed constant) of cold and hot water. T_H and T_C represent the temperatures in the hot and cold compartments, respectively. T_{Hi} and T_{Ci} denote the temperature of the hot and cold inflow, respectively, and V_H and V_C are the volumes of hot and cold water. The temperatures in both compartments evolve according to:

$$V_C \frac{dT_C}{dt} = f_C(T_{Ci} - T_C) + \beta(T_H - T_C)$$
$$V_H \frac{dT_H}{dt} = f_H(T_{Hi} - T_H) - \beta(T_H - T_C)$$

Let the inputs to this system be $u_1 = T_{Ci}, u_2 = T_{Hi}$, the outputs are $y_1 = T_C$ and $y_2 = T_H$, and assume that $f_C = f_H = 0.1$ (m^3/min), $\beta = 0.2$ (m^3/min) and $V_H = V_C = 1$ (m^3).

- Write the state space and output equations for this system in modal form.
- In the absence of any input, determine $y_1(t)$ and $y_2(t)$.
- Is the system BIBO stable? Show why or why not.

Problem 9

BIBO Stability. Consider a single input single output LTI system with transfer function $G(s) = \frac{1}{s^2+1}$. Is this system BIBO stable?

Problem 10

Minimal polynomials. Given a matrix A with minimal polynomial $s^3(s+1)^2(s+2)$, is the system $\dot{x} = Ax$ stable?

Problem 11

Stability. The equations for errors in an inertial navigation system are approximated by:

$$\begin{aligned}\dot{\delta x} &= \delta v \\ \dot{\delta v} &= -g\ \delta\psi + E_A \\ \dot{\delta\psi} &= \frac{1}{R}\delta v + E_G\end{aligned}$$

where δx is the position error, δv is the velocity error, $\delta\psi$ is the tilt of the platform, g is the acceleration due to gravity, and R is the radius of the earth. The input terms are the accelerometer bias E_A and the gyro bias E_G.

1. Is the system internally stable? Is it BIBO stable for any output $y = C[\delta x\ \delta v\ \delta\psi]^T$? Explain.
2. Consider a constant gyro bias $E_G = c$, where c is a constant. Assume no accelerometer bias $E_A = 0$. By deriving an expression for the position error as a function of time, discuss what happens to the position error.